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ORIGINAL ARTICLE

Design of Inequality Models of Covid-19 Disease Incorporating Social Discriminants

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Abstract

Fatal disease like Coronavirus (COVID-19) is a contagious disease which causes death. In 2019 and 2020, millions of deaths were recorded as a result of its outbreak. The impact of the outbreak disrupted Economic and social activities globally. The spread across globe exposed existing social and health inequalities. Disparities in hygiene and level of awareness of the havoc the disease can cause had been the bane of public health interventions. To verify this claim, inequality of non-linear mathematical models were proposed and analyzed to investigate how social determinants influence infection spread and mortality rates. Stability analyses of the model's equilibra were performed. Basic reproduction number was derived for the inequality model R0. Numeric analysis was also carried out to support results. The results revealed significant disparities in the disease's outcomes in heath inequalities indicators.

Keywords

Inequalities, Hygiene, Awareness, Reproduction number stability and Numerical analysis

Introduction

Disease as we all know is a discomfort to human body as well as animals. The spread and hazard of the outbreak of viral diseases cannot be underestimated. It affects virtually everything ranging from economy, to education and health to mention but a few. In 2019, outbreak of COVID-19 ravaged and threatened the existence of the world, the world leaders, health personnel, government officials had great task in curbing and controlling the spread of the disease. Different researchers have contributed in one way or the other to model the spread and understand its impact and how to control the spread and treatment methods to reduce the number of infectives [1] enlarged the frame work of epidemiology by considering the size of epidemic and the duration with a view of finding the probability of disease extinction. The effect of infection period within Susceptible-Infected-Recovered (SIR) models was studied by [2] and discovered that unstable-like behaviour was seen in a finite population in the model. The spatial study of disease spread is important, [3] used spatial study to discover multigroup epidemic in the Susceptible-Exposed-Infected-Recovered (SEIR) model when they extended works of ONeill and Roberts of 1999 on using Monte Carlo Markov Chain to estimate parameters in the model. When there is an outbreak of disease, vaccination against such disease is imminent. Various modeling approaches have been employed to study the spread and impact of COVID-19. Compartmental models, such as the SIR model, have been widely used to simulate disease dynamics and evaluate intervention strategies [4] introduced vaccinated group into the epidemic model to study the non-linear incidence rate in the bifurcation model. If the disease persists in the population then, the stability analysis is expected, [5] introduced migration rate into the susceptible population to study the stability of the model. Within short time of epidemic, a lot can happen, the problems of modelling stochastic epidemic model was looked into by [6,7] captured the transmission dynamics of Measles using stochastic model in the analysis of the spread of the disease. More sophisticated

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models, incorporating socioeconomic and demographic variables, have provided deeper insights into the factors driving health inequalities. Machine learning techniques have also been leveraged to predict COVID-19 outcomes and identify high-risk populations [8] applied non-linear compartmental deterministic mathematical model with exploratory data to discover the extent of the variables: Awareness and hygiene can go in curbing the menace of the spread of COVID-19.

When a disease surfaces in the population, it affects the equality of social status of the populace. To address the issue of inequalities in epidemic modelling, [9] developed mathematical model to study transmission of two vaccine-preventable infections in a population by determining the integration of two social groups. Socioeconomic status (SES) is a critical determinant of health outcomes, including susceptibility to infectious diseases. Research has consistently shown that lower SES is associated with higher risk of infection and adverse health outcomes. During the COVID-19 pandemic, individuals with lower income levels, precarious employment, and inadequate housing have faced higher infection risks due to limited ability to practice physical distancing and greater exposure to public-facing occupations.

Studies have also highlighted how economic deprivation correlates with higher COVID-19 mortality rates, underscoring the urgent need for socioeconomic considerations in public health models. Though, in studying inequalities in epidemic model, the contributing factors in the cases of reported health infractions should be looked into.

The difference in the severity of the diseases as studied by [10,11]. Some affected individuals have different propensity to seek medical care [12]. Geographic location also plays a pivotal role in shaping health inequalities during the COVID-19 pandemic. Urban areas, with their higher population densities, have generally experienced more severe outbreaks than rural areas. However, rural areas often suffer from limited healthcare infrastructure and resources, compounding the challenges faced by residents in these regions. Geographic models of COVID-19 have sought to capture these spatial disparities, providing valuable insights for targeted interventions and resource allocation. Other factors like poverty and social inequalities were discussed in [13]. Also, the reproduction number of the disease are different from region to region [14]. Studies have emphasized the need for health system resilience and preparedness in mitigating the impacts of pandemics on vulnerable populations. In developing countries, level of awareness of the transmission dynamics of infectious disease is very low and the hygiene practice during the outbreak of disease is far from the standard practice which may result into lot of casualties of human lives.

To this end, this research paper looks into the disparities in terms of inequalities of hygiene and level of awareness among susceptible population in a non-linear mathematical model.

Model Description

Considering the dynamics of the virus, we assumed homogeneous population. Also, in the assumption, we assumed disease can only spread if there is a direct contact of an infected person with the susceptible individual. The homogeneous population is subdivided into Susceptible - Quarantined - Infected- Recovered- Exposed - Susceptible (SQIRES) non-linear compartments.

The dynamics goes thus: There is recruitment into susceptible compartment *S(t)* at the rate of $v_x(x)$, after the recruitment and contact with an infected person, those who show the symptoms of the virus are Quarantined *Q(t)* at the rate $\lambda(h)$ which will later move to infected compartment *I(t)* at the rates P_1 and P_2 for those who tested negative. The recovered individuals R(t) are populated at the rate of P_3. From the fact that individuals will exit the system either naturally or induced, the natural death rate and disease induced death rate are respectively given as σ and δ respectively. The exposed compartment E(t) is used for those that have information about the virus (awareness) and this group are defined thus: It comprises of individuals who maintain personal hygiene and have attained certain level of awareness which we believe will reduce the spread rate of the virus. This rate is described thus

$$\Omega(G) = \Omega_{\max} - G\left(\Omega_{\max}\right) - G\left(\Omega_{\min}\right) \tag{4.1}$$

where Ωmax and Ωmin represent the maximum and minimum transmission rates of the virus respectively.

This is important in studying the dynamics of disease spread in that, if improved sanitation behaviour and personal hygiene are encouraged, these will definitely play critical roles in bringing down the spread of the virus from all forms of transmission. So in the event of disease outbreak such as COVID-19, frequent hand washing and use of sanitizers are encouraged as first precautionary measures. These inform that the spread of the disease can be effectively controlled as fast as possible.

This equation (4.1) can be reduced if maximum hygiene level (G) is assumed to be 1, then equation (4.1) will be

$\Omega(G) = \Omega_{min}$ which clearly showed that the disease transmission can be reduced to the barest minimum if some level of hygiene is reached and vice-versa if G = 0. To this end, having showed that the transmission rate of COVID-19 can be reduced if some level of hygiene is achieved and a campaign strategy to educate and create awareness for the locals are put in place. Let us assumed that a certain fraction G_0 of the population practice and have good habits in terms of sanitation before the outbreak of COVID-19, that means the remaining individuals in the population denoted by $(1\text{-}G_0)$ will be highly spirited to practice healthy sanitation habits as the rate of education campaign on good hygiene grows at rate ϑ. On the spur of moment of disease outbreak, thus, as the rate of awareness on what to do increases, it is assumed the spread of the disease reduces.

$$G(x) = \frac{G_0(x) + E(x) \times G_0(x) + \eta_0 \times E(x) - G(x) \times \eta_0 \times E(x)}{1 + E(x)}, G_0(x) \le G(x) \le 1 \quad (4.2)$$

Now, in equation (4.2), it shows that, if $E(x) = 0$ that is no strategy inform of campaign to advise populace on how to go about protecting themselves against the disease, then $G = G_0$. However, in the long run, if a well planned and awareness campaign strategy to disseminate information on how individuals can combat or contain the spread, there will be an increase in sanitation initial level from G_0 to $G_0 + (1 - G_0)\eta_0$.

Suppose, the chance that an individual becomes infected due to individual's interaction with an infected and infectious human is r_1, then an assumption of the force of infection can be put as $\lambda(G)$ which simply follows a mass action and defined thus as

$$\lambda(G) = r_1 \times \Omega(G) \times I(x) \quad (4.3)$$

In the construction of the inequality model, the following assumptions were made:

The transmission of COVID-19 is possible only if an infected or an infectious human come in contact with individuals; or by individuals coming in contact with surface which has been infested with the virus.

There is a saturating function that propagates awareness of the havoc of the virus and how susceptible individual could protect himself and this depends on infected population density given by

$$F(I) = \frac{\eta_0 \times I(x)}{\eta_1 + \eta_2 \times I(x)} \quad (4.4)$$

In [15], equation (4.4), that is the saturating function was used also where η_0, η_1 and η_2 depicted rate of information growth which is the half of saturation point, F(I) attains half of its maximum value $\frac{\eta_0}{\eta_2}$ when infected population arrives at η_1 and saturation constant of information respectively [15,16].

We also assume that there is no permanent immunity that is, recovered individual over time from COVID-19 lose his immunity δ (whether natural or due to medication) and moves back to the susceptible compartment.

We assume that awareness or information degenerate at rate a. This assumption is due to the response rate behaviour of humans to disease outbreak in the long run.

Since we have varying population with time, the recruitment rate into the population is assumed to be

$$\upsilon(x) \ge \varpi \int (0,1] \upsilon x(x) dx + \int (0,1] \upsilon \dot{o}(x) dx, \varpi \in [0,1) \quad (4.5)$$

where $\upsilon_x(x)$ are those who are recruited through migration, $\upsilon_\epsilon(x)$ are rate of births while $\varpi \in (0,1]$ is a restriction parameter on immigration.

Model Formulation

Based on the above assumptions and using the following description of the variables of system of equations used.

(i) $S(t)$ represents Susceptible humans.

(ii) $Q(t)$ represents Quarantined humans.

(iii) $I(t)$ represents Infectious humans.

(iv) $R(t)$ represents Recovered humans.

(v) $E(t)$ represents Education/level of hygiene.

Hence, the dynamics of the model is presented in the following system inequalities of non-linear compartmental differential equations:

$$
\begin{aligned}
d\left((x)\right) &\geq \int \bar{A}dx - \lambda(G)\int S(x)dx - \sigma\int S(x)dx + \rho 2\int Q(x)dx + \delta\int R(x)dx \\
d\left(Q(x)\right) &\geq \lambda(G)\int S(x)dx - \left(\rho_1 + \rho_2 + \sigma\right)\int Q(x)dx \\
d\left(I(x)\right) &\geq \rho_1\int Q(x)dx - \left(\rho 3 + q + \sigma\right)\int I(x)dx \\
d\left(R(x)\right) &\geq \rho_3\int I(x)dx - \left(\delta + \sigma\right)\int R(x)dx \\
d\left(E(x)\right) &\geq \eta_0\int \frac{I(x)}{\eta 1 + \eta_0 \times I(x)} d(x) - a \times d\left(E(x)\right)
\end{aligned}
\tag{5.1}
$$

Since the model involves human dynamics, it is assumed that all parameters and variables used are positive.

Model Analysis

Our focus here, is how to analyse the model. First, we perform stability analysis of the model viz the positivity and boundedness analysis, the steady states of the model and their stability.

Positivity and Boundedness of solutions

Here, we show that system (5.1) is epidemiological well-posed and is realistic if all the system variables of (5.1) are positive for all time *t*. With this in mind, we establish the claim using the following Corollaries.

Corollary 6.1

Suppose S(x), Q(x), I(x), R(x) and E(x) are non positive functions.

Let $S(0) \geq 0$, $Q(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$, $G(0) \geq 0$ and S(0) be non-negative while $x > 0$.

The rates above are bounded in plane $\Psi \in \square^5$.

Proof. In [15] we have the following

$$
\begin{aligned}
d\left(S(x)\right) &\geq \bar{A} + \rho 2\int^1 0Q(x)\,dx + \delta\int^1 0R(x)dx \\
d\left(Q(x)\right) &\geq \lambda\int_0^1 S(x)dx \\
d\left(I(x)\right) &\geq \rho 1\int_0^1 Q(x)dx \\
d\left(R(x)\right) &\geq \rho 3\int_0^1 I(x)dx \\
d\left(E(x)\right) &\geq \eta_0\int_0^1 \frac{\eta 0 \times I(x)}{\eta_1 + \eta_2 \times I(x)}dx
\end{aligned}
\tag{6.1}
$$

It will be convenient to prove that the system (5.1) region is positive, not variant and attractive [17,18]. The region of attraction of (6.1) becomes

$$
\Psi \in \left\{\left(S(x), Q(x), I(x), R(x)E(x)\right) \in \square^5 : S(x) + Q(x) + I(x) + R(x) \leq \frac{\bar{A}}{\sigma}, E(x) \leq \frac{\eta}{a}\right\} \tag{6.2}
$$

It is also sufficient to study the dynamics of inequalities above in which all solutions originating in the inside positive orphan can be drawn [15].

Equilibrium points of System (5.1)

Since system (5.1) is inequalities of non-linear and its exact solutions may not be easy to determine. Consequently, knowledge of stability theories is needed here to investigate the qualitative behaviour of the equilibrium points so that we can have insight about the disease dynamics and how the spread can be contained. Hence, the existence of all equilibrium of system (5.1) are thoroughly investigated by making the rate of change with respect to time *t* of all the non-linear inequalities variables zero. Assume *G*(*x*) is a quantity that its value is known, then the equilibrium points of system (5.1) can be obtained by solving simultaneously the set of algebraic inequalities:

$$\begin{aligned}
&\overline{A}-\lambda(G)\times S(x)-\sigma\times S(x)+\rho_2\times Q(x)+\delta\times R(r)\geq 0\\
&\lambda(G)\times S(x)-\rho_1\times Q(x)-Q(x)\times\rho_2-Q(x)\times\sigma\geq 0\\
&\rho_1\times Q(x)-\rho_3\times I(x)-q\times I(x)-\sigma\times I(x)\geq 0\\
&\rho_3\times I(x)-\delta\times R(x)-\sigma\times R(x)\leq 0\\
&\frac{\eta_0\times I(x)}{\eta_1+\eta_0\times I(x)}\geq\frac{d(E(x))}{dx}
\end{aligned}\tag{7.1}$$

Simplifying first and last inequalities of system (6.1) yield

$$\begin{aligned}
S(x)&\geq\frac{(\rho_1+\rho_2+\sigma)(\rho_3+q+\sigma)(\delta+\sigma)\times\overline{A}}{A_0\times\lambda(G)+A_1}\\
Q(x)&\geq\frac{(\rho_3+q+\sigma)(\delta+\sigma)\times\overline{A}\times\lambda(G)}{A_0\times\lambda(G)+A_1}\\
I(x)&\geq\frac{\rho_1\times(\delta+\sigma)\overline{A}\times\lambda(G)}{A_0\times\lambda(G)+A_1}\\
R(x)&\geq\frac{\rho_1\times\rho_3\times\overline{A}\times\lambda(G)}{A_0\times\lambda(G)+A_1}\\
E(x)&\geq\frac{\eta_0\times\rho_1\times\delta\times\overline{A}\times\lambda(G)+\eta_0\times\rho_1\times\sigma\times\overline{A}\times\lambda(G)}{A_2\times\lambda(G)+A_3}
\end{aligned}\tag{7.2}$$

We define

$$\begin{aligned}
A_0(x)&\geq\rho_1\times\rho_3\times\sigma+\rho_1\times(\delta+\sigma)(q+\sigma)+\sigma\times(\delta+\sigma)(\rho_3+q+\sigma)\\
A_1(x)&\geq\sigma\times(\delta+\sigma)\times(\rho_1+\rho_2+\sigma)\times(\rho_3+q+\sigma)\\
A_2(x)&\geq a\times(\eta_1\times A_0+\eta_2\times\rho_1\times(\delta+\sigma)\times\lambda)\\
A_3(x)&\geq a\times\eta_1\times A_1
\end{aligned}\tag{7.3}$$

Merging inequality (7.3) of *I(x)* in later with (4.3) gets

$$\lambda(G)\geq\frac{r_1\times\delta\times\rho_1\times\Omega(G)\times\overline{A}\times\lambda(G)+r_1\times\delta\times\sigma\times\Omega(G)\times\overline{A}\times\lambda(G)}{A_0\times\lambda(G)+A_1}\tag{7.4}$$

which have the following quadratic solutions

$$\lambda(G)\geq 0\ or\ \lambda(G)\geq\frac{r_1\times\rho_1\times\delta\times\overline{A}\times\Omega(G)+r_1\times\rho_1\times\sigma\times\overline{A}\times\Omega(G)-A_1}{A_1}\tag{7.5}$$

Instance 1:

It was observed that (7.5) shows two interpretation

First interpretation of result i.e.,

$$\lambda(G)\geq 0$$

and second interpretation

$$\lambda(G)\geq\frac{r_1\times\rho_1\times\delta\times\overline{A}\times\Omega(G)+r_1\times\rho_1\times\sigma\times\overline{A}\times\Omega(G)+A_1}{A_1}\tag{7.6}$$

COVID-19 Free Equilibrium, $P_0(x)$ in (4.3), we note that when putting $\xi(G) = 0$, we have

$$P_0(x)\geq\left(\frac{\overline{A}}{\sigma},0,0,0,0\right)\tag{7.7}$$

COVID-19 Free equilibrium is defined as $P_0(x)$ and shows the point at which the model is free of any infection of novel Corona virus disease. With this, the infection can be controlled without hygiene in the entire population area.

Local Stability of COVID-19 Free Equilibrium (CFE), P_0

The local stability of P_0 is examined in the theorem below:

Theorem A:

The COVID-19 Free Equilibrium (CFE). $P_0(x)$ in (??) is considered to be locally asymptotically stable if $D_0 < 1$, if not is unstable.

Proof. Apply Jacobian matrix to (7.8) at $P_0(x)$ in the absence of sanitation $G(x) = G_0(x)$

$$J\left(P_0(x)\right) \geq \begin{pmatrix} -\sigma & \rho_2 & -\dfrac{r_1 \times \Omega(G_0) \times \bar{A}}{\sigma} & \delta & 0 \\ 0 & -(\rho_1 + \rho_2 + \sigma) & -\dfrac{r_1 \times \Omega(G_0) \times \bar{A}}{\sigma} & 0 & 0 \\ 0 & \rho_1 & -(\rho_3 + q + \rho_2) & 0 & 0 \\ 0 & 0 & \rho_3 & -(\delta + \sigma) & 0 \\ 0 & 0 & \dfrac{\eta_0}{\eta_1} & 0 & -a \end{pmatrix} \tag{7.8}$$

It is interesting to note that eigenvalues λ of (??) has the following values.

$$\lambda_1 = -\sigma$$

$$\lambda_2 = -(\delta + \sigma)$$

$$\lambda 3 = -a$$

In the matrix (7.8) above, we need to get the other two eigenvalues using the following two by two matrix

$$M(x) \geq \begin{pmatrix} (-\rho_1 + \rho_2 + \sigma) & -\dfrac{r_1 \times \Omega(G_0) \times \bar{A}}{\sigma} \\ \rho_1 & -(\rho_3 + q + \rho_2) \end{pmatrix} \tag{7.9}$$

Going by Routh-Hurwitz condition, the eigenvalues of matrix M are real and negative if

(i) Trace (M) < 0

(ii) Determinant (M) > 0

Tr(M) = $-(\rho_1 + \rho_2 + \rho_3 + 2\sigma) < 0$

$$Det(M) = \frac{(\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma) - \rho_1 \times r_1 \times \Omega \times (G_0) \times \bar{A}}{\sigma}$$

$$= \frac{(\sigma \times (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma))(\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma) - \sigma \times (\rho_1 - \rho_2 + \sigma)(\rho_3 + q + \sigma) - \rho_1 \times r_1 \times \Omega(G_0) \times \bar{A}}{\sigma \times (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)}$$

$$= (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)(1 - D_0) > 0 \; if \; D_0 < 0$$

Where

$$D_0(x) = \left(\frac{\rho_1 \times r_1 \times \Omega(G_0) \times \bar{A}}{\sigma \times (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)} \right) \tag{7.10}$$

Thus, all eigenvalues found are real and negative if $D_0 < 1$ so that CFE, P_0 is locally asymptotically stable and unstable if $D_0 > 1$.

Remark 1

The quantity D_0 in equation (7.10) is generally referred to as the basic reproduction number. This is the average number of secondary infected humans that an infected human can infect due to direct or indirect contact in his/her infectious period in a wholly susceptible population. In particular if $D_0 < 1$ (or $D_0 > 1$), it reflects that on the average, an infected individual will infect successfully less than (or more than) one secondary infected individuals in a population that is wholly susceptible during his/her whole infectious period and thus respectively, the disease dies out (or persists) in the population. It is worthy of note that the derivation of the basic reproduction number R_0 can be effectively obtained by making use of the next generation matrix approach [19,20].

Global asymptotic stability of CFE, P_0

We further probe the asymptotic stability of P_0 by constructing the Lyapunov function for Global Aymptotic Stability (GAS). Thus, consider the Lyapunov function defined as follows:

$$L\left(Q(x), I(x)\right) = \rho_1 \times Q(x) + \rho_1 \times I(x) + \rho_2 \times I(x) + \sigma \times I(x) \tag{7.11}$$

Differentiate (7.11) with respect to *x* along the solution of (??) yields

$$\begin{aligned}
&\frac{d\left(L\left(Q(x), I(x)\right)\right)}{dx} = \rho_1 \times \left(\lambda(G) \times S(x) + (\rho_1 + \rho_1 + \sigma) + (\rho_1 + \rho_2 + \sigma)\left(\rho_1 Q(x) + (\rho_3 + q + \sigma)\right)\right) \\
&= \rho_1 \Omega(G) \times S(x)(\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma) I(x) \\
&= \rho_1 r_1 \lambda(G) \times S(x) \times I(x) - (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma) \times I(x) \\
&= \left(\rho_1 r_1 \lambda(G) \times S(x) - (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)\right) \times I(x)
\end{aligned} \tag{7.12}$$

putting $G = G_0$, $S = \dfrac{\overline{A}}{\sigma}$ in (7.12) obtain:

$$\begin{aligned}
&\frac{d\left(L\left(Q(x), I(x)\right)\right)}{dx} = \left(\rho_1 \times r_1 \times \Omega(G_0)\frac{\upsilon}{\sigma} - (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)\right) \times I(x) \\
&= (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)\left(\frac{\rho_1 \times r_1 \Omega(G_0)\overline{A}}{(\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)} - 1\right)
\end{aligned} \tag{7.13}$$

and

$$\frac{d\left(L\left(Q(x), I(x)\right)\right)}{dx} = (\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma) \times (D_0 - 1) \times I(x) \geq 0 \; if \; D_0 \geq 1$$

Therefore, the CFE, P_0 is globally asymptotically stable if $D_0 \geq 1$, if not unstable. The foregoing considerations are summarized thus.

Theorem B:

The CFE, P_0 is asymptotically stable globally if $D_0 \geq 1$, if not unstable. If

$$\lambda(G) \geq \frac{r_1 \times \rho_1 \times \delta \times \overline{A} \times \Omega(G) + r_1 \times \rho_1 \times \sigma \times \overline{A} \times \Omega(G) + A_1}{A_1}$$

COVID-19 endemic equilibrium (CEE), D_1. The Dynamical components of (??) which are represented with D_1

$$\begin{aligned}
&P_1 = \left(S_1(x), Q_1(x), I_1(x), R_1(x) E_1(x)\right) \\
&= \left(\frac{(\rho_1 + \rho_2 + \sigma)(\rho_3 + q + \sigma)(\delta + \sigma) \times \overline{A}}{A_0 \times \lambda(G) + A_1}\right), \frac{(\rho_3 + q + \sigma)(\delta + \sigma) \times \overline{A} \times \lambda(G)}{A_0 \lambda(G) + A_1}, \\
&\frac{\rho_1 \times (\delta + \sigma) \times \overline{A} \times \lambda(G)}{A_0 \times \lambda(G) + A_1}, \frac{\rho_1 \times \rho_3 \times \overline{A} \times \lambda(G)}{A_0 \times \lambda(G) + A_1}, \frac{\rho_1 \times \delta \times \overline{A} \times \lambda(G) + \rho_1 \times \sigma \times \overline{A} \times \lambda(G)}{A_3 \times \lambda(G) + A_3} \geq 0
\end{aligned} \tag{7.14}$$

P_0 represents the Coronavirus Endemic Equilibrium (COVID-19 EE) which represents a situation whereby there is a presence of Coronavirus disease or infection in the population. Assuming *G* is known, then, there exists at most one endemic equilibrium P_0 as defined in (??). In order to show that (??) indeed specifies an endemic equilibrium, we must not fail to show that

$$\rho_1 \times \overline{A} \times \Omega(G) \times \delta + \rho_1 \times \sigma \times \overline{A} \times \Omega(G) \geq A_1 \tag{7.15}$$

furthermore from (7.15)

$$\Omega(G) = \frac{A_1}{\rho_1 \times r_1 \times \delta \times \overline{A} + \rho_1 \times r_1 \times \sigma \times \overline{A}} \tag{7.16}$$

Substitute for (4.1) in (7.16) the subject gets

$$\Omega(G) \geq \frac{\rho_1 \times r_1 \times \delta \times \overline{A} \times \Omega_{\max} + \rho_1 \times r_1 \times \sigma \times \overline{A} \times \Omega_{\max} - A_1}{\rho_1 \times r_1 \times \delta \times \overline{A} \times \Omega_{\max} + \rho_1 \times r_1 \times \sigma \times \overline{A} \times \Omega_{\max} - \rho_1 \times r_1 \times \delta \times \overline{A} \times \Omega_{\min} - \rho_1 \times r_1 \times \sigma \times \overline{A} \times \Omega_{\min}} \tag{7.17}$$

$$\Omega(G) \le \frac{\rho_1 \times r_1 \times \delta \times \bar{A} \times \Omega_{\max} + \rho_1 \times r_1 \times \sigma \times \upsilon \times \Omega_{\max} - A_1}{\rho_1 \times r_1 \times \delta \times \bar{A} \times \Omega_{\max} + \rho_1 \times r_1 \times \sigma \times \bar{A} \times \Omega_{\max} - \rho_1 \times r_1 \times \delta \times \bar{A} \times \Omega_{\min} - \rho_1 \times r_1 \times \sigma \times \upsilon \times \Omega_{\min}} \quad (7.18)$$

To complete **Theorem A:**

We need the following Corollary:

Corollary A:

Suppose *G(x)* is known, then system (??) has a unique endemic equilibrium $P_1(x)$ that is positive if the force of infection is

$$\lambda(G) \ge \frac{r_1 \times \rho_1 \times \delta \times \bar{A} \times \Omega(G) + r_1 \times \rho_1 \times \sigma \times \bar{A} \times \Omega(G) + A_1}{A_1(x)} \ge 0 \quad (7.19)$$

Resolving for *G(x)* we need to study the endemic equilibrium P_1 completely, it is necessary to get equation for *G(x)* and find its uniqueness. The equation for *G(x)* is obtained by making $G(x) = G_1(x)$ in (6.2) to obtain

$$\lambda(G) \ge G_0 + (1 - G_0)\eta_0 \frac{\dfrac{\eta_0 \times \rho_1 \times \delta \times \bar{A} \times \lambda(G) + \sigma \times \bar{A} \times \lambda(G)}{A_2 \times \lambda(G) + A_3}}{1 + \dfrac{\eta_0 \times \rho_1 \times \delta \times \bar{A} \times \lambda(G) + \sigma \times \bar{A} \times \lambda(G)}{A_2 \times \lambda(G) + A_3}} \quad (7.20)$$

$$= G_0 + \frac{\left(\eta^2 \times \rho_1 - \eta^2 \times \rho_1 \times G_0(x)\right)\left(\delta \times \bar{A} \times \lambda(G) + \sigma \times \bar{A} \times \lambda(G)\right)}{\left(A_2 \times \lambda(G) + \eta_0 \times \rho_1 \bar{A} \times \lambda(G) + \eta_0 \times \rho_1 \times \bar{A} \times \lambda(G)\right)} \ge 0$$

However, for

$$\zeta(G) = \frac{A_5 \times \lambda(G)}{A_5 \times \lambda(G) + A_3} - \left(G(x) - G_0(x)\right) \ge 0 \quad (7.21)$$

Where

$$R_4 = \left(\eta_0 \times \rho_1 \times \bar{A} \times \delta - G0(x) \times \eta_0 \times \rho_1 \times \bar{A} \times \delta + \eta_0 \times \rho_1 \times \bar{A} \times \sigma - G_0(x) \times \eta_0 \times \rho_1 \times \bar{A} \times \sigma\right)$$

$$A_5 = A_2 + \eta_0 \times \rho_1 \times \bar{A} \times \sigma + \eta_0 \times \rho_1 \times \delta \times \sigma \times \bar{A}$$

$$\zeta(G_0) \le 0 \quad (7.22)$$

$$\zeta(G_1) \ge 0 \quad (7.23)$$

$$\int [G0, G_1]\, \zeta(G)\, dx \ge \int [G0, G_1]\, dx \quad (7.24)$$

Combining $G_0(x) = G_1(x)$

$$\zeta(G_0) \ge \frac{A_4(x)\,\lambda(G_0)}{A_5 \times \lambda(G) + A_3} = \frac{r_1 \times \rho_1 \times \Omega(G_0) \times \bar{A} \times \delta + r_1 \times \rho_1 \times \Omega(G_0) \times \bar{A} \times \sigma - A_1}{A_0} \quad (7.25)$$

$$\frac{A_1 \times D_0 - A_1}{A_0} A_0 \ge 0$$

The results following $g(G) \ge 0$ if $D_0 \ge 0$. Therefore, if $G_0(x) = G_1(x)$ in (7.22), $g(G_1) \le 0$ implies

$$\zeta(G_1) \ge -G_1 + G_0 \le 0$$

Differentiate (7.20) with respective to *G(x)* gets

$$\frac{d\zeta(G)}{dx} \ge \frac{A_3 \times A_4 \times \dfrac{d\lambda(G)}{dx} - A^2 5 \times \lambda^2(G) - 2 \times A5 \times \lambda(G) - A^2 3}{A^2 5 \times \lambda^2(G) + 2 \times A_5 \times \lambda(G) + A_3^2} \ge 0 \quad (7.26)$$

We observed that

$$\frac{d\lambda(G)}{dx} \ge -\frac{\rho_1 \times r_1 \times \delta + \rho_1 \times r_1 \times \sigma \times \bar{A} \times \Omega_{\max} - \Omega_{\min}}{A_0} \quad (7.27)$$

The solution of (7.20) in (G_0, G_1) achievable if $D_0 \ge 0$.

Corollary C:

The system (eq 5.1) has a positive unique endemic equilibrium P if $G \in (G_0, G_1)$ and if $D_0 \geq 1$.

Local stability of COVID-19 EE, *P*.

Theorem 3:

The COVID-19 EE, *P* is asymptotically stable locally if the following inequality holds if not, unstable.

$$\lambda(G) \geq \frac{(\rho_1+\rho_2+\sigma)(\rho_3+q+\sigma)}{\rho_1\times\delta\times\upsilon+\rho_1\times\sigma\times\upsilon} \tag{7.28}$$

Take the Jacobian matrix of the (??) at *P* is defined as

$$J(P)=\begin{pmatrix} -A_6-\sigma & \rho_2 & -A_7 & \delta & 0 \\ A_6 & -\rho_1-\rho_2-\sigma & A_7 & 0 & 0 \\ 0 & \rho_1 & -\rho_3-q+\sigma & 0 & 0 \\ 0 & 0 & \rho_3 & -\delta-\sigma & 0 \\ 0 & 0 & A_8 & 0 & -a \end{pmatrix} \tag{7.29}$$

Where

$$A_6=\frac{r_1\times\rho_1\times\Omega(G_0)\times\bar{A}\times\delta+r_1\times\rho_1\times\Omega(G_0)\times\bar{A}\times\sigma-A_1}{A_0}$$

$$A_7=\frac{(\rho_1+\rho_2+\sigma)\times(\rho_3+q+\sigma)}{\rho_1+\sigma+\bar{A}+\rho_1+\delta+\bar{A}} \tag{7.30}$$

$$A_8=\frac{t_0\times A^2 0\times r_1\times\Omega^2(G)\times h_1}{\left((\rho_1\times\bar{A}\times\delta\times h+\rho_1\times\bar{A}\times\sigma\times h+A_0\times h_1)\times r_1\times\Omega(G)-A_1\times h_2\right)^2}$$

The eigenvalues is $\lambda_1 = -a$ and other eigenvalues is derived from

$$M(x)=\begin{pmatrix} -A_6-\sigma & \rho_2 & -A_7 & \delta \\ A_6 & -\rho_1-\rho_2-\sigma & A_7 & 0 \\ 0 & \rho_1 & -\rho_3-q-\sigma & 0 \\ 0 & 0 & \rho_3 & -\delta-\sigma \end{pmatrix} \tag{7.31}$$

We perceive that Trace *(M(x)) < 0* from Corollary 3:

Trace *(M(x))* $= -A_6 - 4\sigma - \rho_1 - \rho_2 - \rho_3 - q - \delta \geq 0$

$$\begin{aligned} Det(M) &= \delta\times\sigma^3+\delta\times\sigma^2\times q+\delta\times\sigma^2\times A_6+\delta\times\sigma^2\times\rho_1+\delta\times\sigma^2\times\rho_2+\delta\times\sigma^2\times\rho_3+\delta\times\sigma\times q\times A_6 \\ &+\delta\times\sigma\times q\times\rho_1+\delta\times\sigma^2+q+\rho_2+\delta\times\sigma^2\times A_6\times\rho_1+\delta\times\sigma\times A_6\times\rho_3-\delta\times\sigma\times A_7\times\rho_1 \\ &+\delta\times\sigma\times\rho_1\times\rho_3+\delta\times\sigma\times\rho_2\times\rho_3+\delta\times q\times A_6\times\rho_1+\sigma^4+\sigma^3\times A_6+\sigma^3\times\rho_1+\sigma^3\times\rho_2+\sigma^3\times\rho_3 \\ &\sigma^2\times q\times A_6+\sigma^2\times q\times\rho_1+\sigma^2\times q\times\rho_2+\sigma^2\times A_6\times\rho_1-\sigma^2\times A_7\times\rho_1+\sigma^2\times\rho_1\times\rho_3+\sigma^2\times\rho_2\times\rho_3 \\ &+\delta\times\sigma^2\times q+\delta\times\sigma^2\times A_6 \\ &=\sigma\times\rho_1\times(A_6-A_7)\times(\delta+\sigma)+\sigma^2\times\sigma^2\times A_6+\sigma^2\times\rho_2\times\rho_1+\sigma^2\times\rho_2\times\rho_3+\delta\times\sigma^2\times q+\delta\times\sigma^2\times A_6 \\ &+\delta\times\sigma^2\times\rho_1+\delta\times\sigma^2\times\rho_2+\delta\times\sigma^2\times\rho_3+\sigma^4+\sigma^3\times\rho_2+\sigma^3\times\sigma^3+q\times\sigma^3+\sigma^3\times A_6+\sigma^3\times\rho_1 \\ &+\delta\times\rho\times q\times A_6+\delta\times\rho\times q\times\rho_1+\delta\times\rho\times q\times\rho_2+\delta\times\rho\times A_6\times\rho_1+\delta\times\sigma\times\rho_1\times\rho_2+\delta\times\sigma\times\rho_2\times\rho_3 \\ &+\delta\times q\times A_6\times\rho_1+\sigma\times q\times A_6\times\rho_1+\sigma\times A_6\times\rho_1\times\rho_3 \end{aligned} \tag{7.32}$$

If

$$A_6-A_7\geq\frac{r_1\times\rho_1\times\Omega(G)\times\bar{A}\times\delta+r_1\times\rho_1\times\Omega(G)\times\bar{A}\times\sigma-A_1}{A_0}-\frac{(\sigma+\rho_1\times\rho_2)\times(\sigma+q\times\rho_3)}{\rho_1\times\sigma\times\bar{A}+\rho_1\times\delta+\bar{A}}$$

If $\lambda(G)\geq\frac{(\sigma+\rho_1\times\rho_2)\times(\sigma+q\times\rho_3)}{\rho_1\times\sigma\times\bar{A}+\rho_1\times\delta+\bar{A}}$

Then, *Det(M)* ≥ 0.

Therefore, D_0 is locally asymptotically stable.

Sensitivity Analysis of system (5.1)

This is to determine the key parameters that can throw system 5.1 into an endemic situation. That is, calculating the parameters shows the relative change of the model if the parameters change slightly. Using [21,22] approach,

$$\frac{y}{D_0}\times\frac{\partial D_0}{\partial y} \tag{8.1}$$

where y represents each parameter in the model we need to find its sensitivity, and with the meaning of D_0. The sensitivity of the parameters is calculated thus:

The sensitivity of the following parameters in the model as given in section 3, Ω_{Max}, *r*1, *σ*1, *γ*, *αt*, *αc* and others are investigated thus for example:

$$\frac{\Omega_{Max}}{D_0}\times\frac{\partial D_0}{\partial \Omega_{Max}}\geq 0 \tag{8.2}$$

which is positive. All other parameters are investigated in like manner. We observed also that *r1, γ, α1, p1,T* and B are also positive and sensitive to D_0. The parameters that are positive like Ω_{Max}, *r1* and *γ* will definitely increase and bring about the same proportion in D_0.

Sensitivity analysis of G_0, ρ_1, ρ_3 and, *σ* showed a relationship that is inversely proportional to D_0. This implies that an increase in *H0, α2, α3, s* and will bring a reduction in the value of D_0. However, in the real sense, this is not practicable except for a2. The implication of the sensitivity analysis results suggest that more efforts in particular should be geared towards reducing the risk of contracting the disease represented by r_1 and the transmission rate represented by Ω_{Max}.

Specifically, if *γ* = 0 that is, the rate at which we have immigrants is reduced to zero, this means the border are closed completely to immigrants, this will in turn reduce the basic reproduction number (D_0) will be but if the border is completely opened to immigrants then (D_0 = 1), this means D_0 will increase. Here, if the disease is communicable, that means, the probability of disease contraction by an individual (p1), rate of transmission per day Ω_{Max} (a1) which is also the rate at which quarantined individuals progress to the infected compartment are greater than zero (i.e, $r_1 > 0$, $\Omega_{Max} > 0$ and $\rho_1 > 0$). Hence, there is a problem at hand for the health workers to deal with. This scenario poses a pointer to the fact that public and international health workers should ensure that transmission rate Ω_{Max}, p1 and a1 are kept relatively low because the parameters are capable of increasing reproduction number D_0. Keeping the aforementioned parameters low, we believe, through various awareness programme or propagation of information on how to curb the disease transmission and protection strategies of individuals such as maintaining good hygiene and placing restriction through various social media available will go a long way in curbing the spread of the disease. To achieve this feat, Government can institute test centres and create isolation centres for those who come into the country during the outbreak while on the part of individuals within the locality of the outbreak who are not infectious maintain good hygiene and be disciplined about it. This will ensure individuals that test positive are moved into isolation centres for further test, examination and treatment. Those who test negative will be allowed to progress into susceptible population.

Results of Analysis

The numerical simulations with different scenarios are presented thus: Figure 1 shows income distribution scenario, which showed the majority of susceptible individuals are low income earners as such, may not have access to health facilities in developing countries. The income distribution shows a normal distribution centered around the mean income. This provides a baseline for understanding the socioeconomic spread within the population.

Significant disparities are visible, with minority groups showing higher infection and mortality rates. This can be attributed to systemic inequalities such as healthcare access, occupational risks, and socioeconomic status. Ethnicity viz-a-vis infection rate and mortality scenario as presented in Figure 2 and Figure 3.

The disparities in the ethnic population showed that the more the population the more the risk of contracting the virus which in turn brings about increase in mortality as depicted in Figure 3.

In Figure 4, the transmission of the virus increased with the low income group and reduces as the income group becomes higher. Figure 5 depicts the mortality rate and region, it was discovered that the mortality rate was higher in the urban region than that of rural region because of over concentration of individuals. Urban areas might show higher infection and mortality rates compared to rural areas due to higher population density, increased person-to-person contact, and potentially overburdened healthcare systems in cities. The plot likely shows a negative correlation, indicating that lower income levels are associated with higher infection rates. This suggests that lower-income individuals might have higher exposure due to essential jobs, crowded living conditions, or limited access

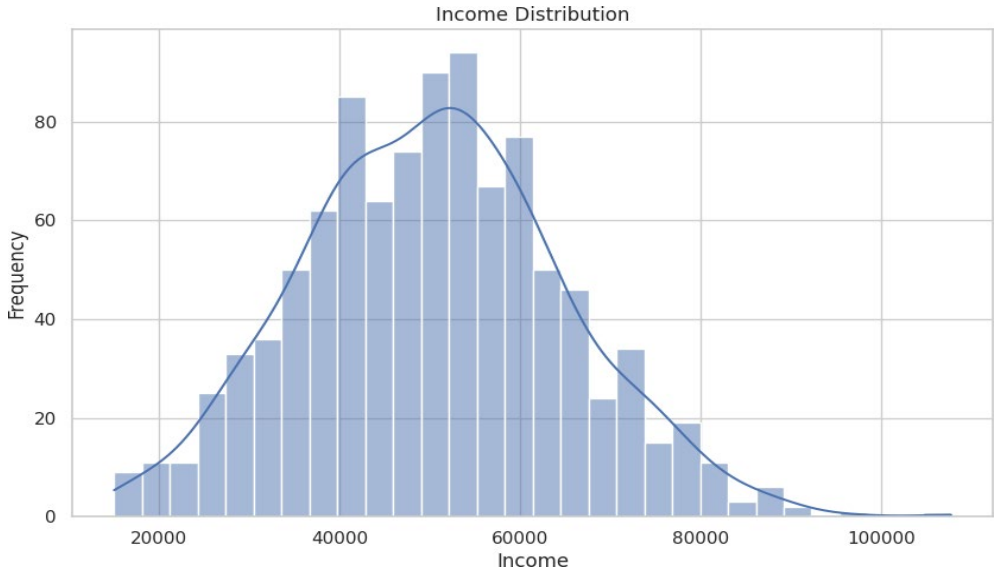

Figure 1: Income distribution.

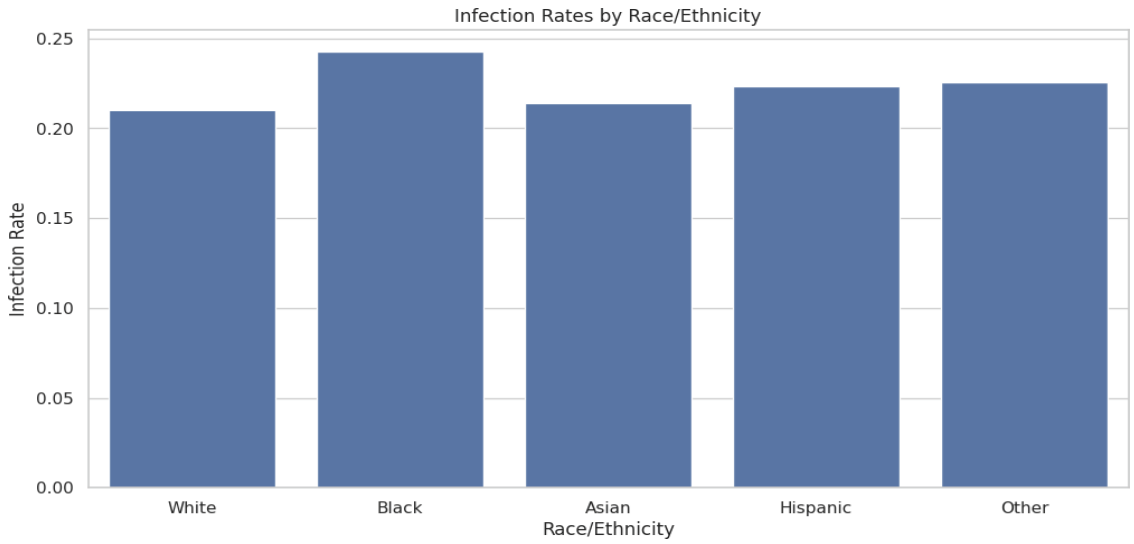

Figure 2: Infection rates by ethnicity.

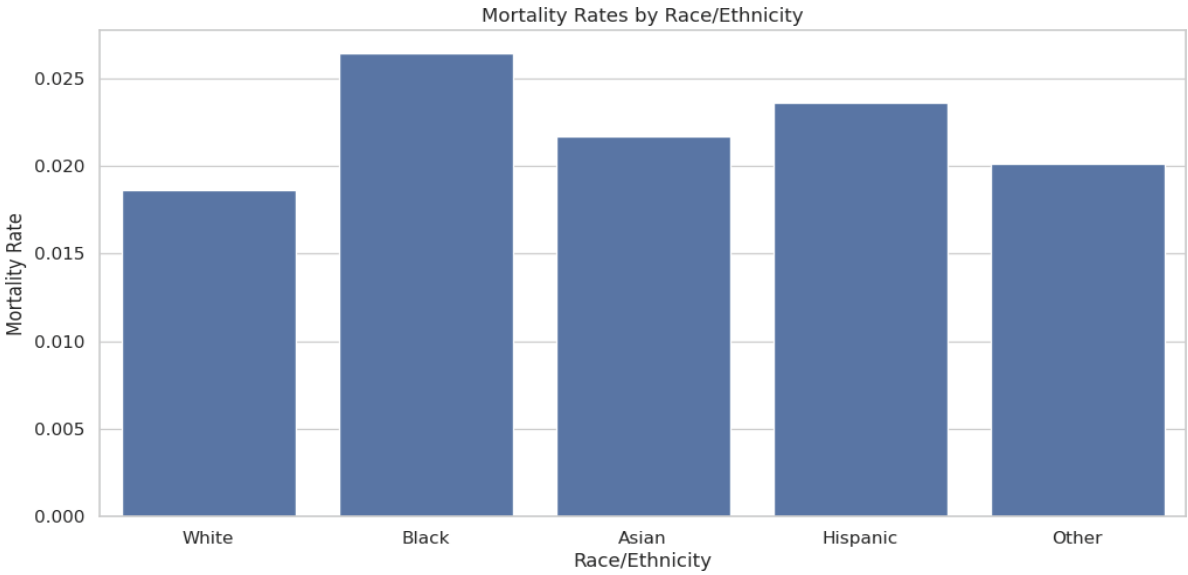

Figure 3: Mortality rate by ethnicity.

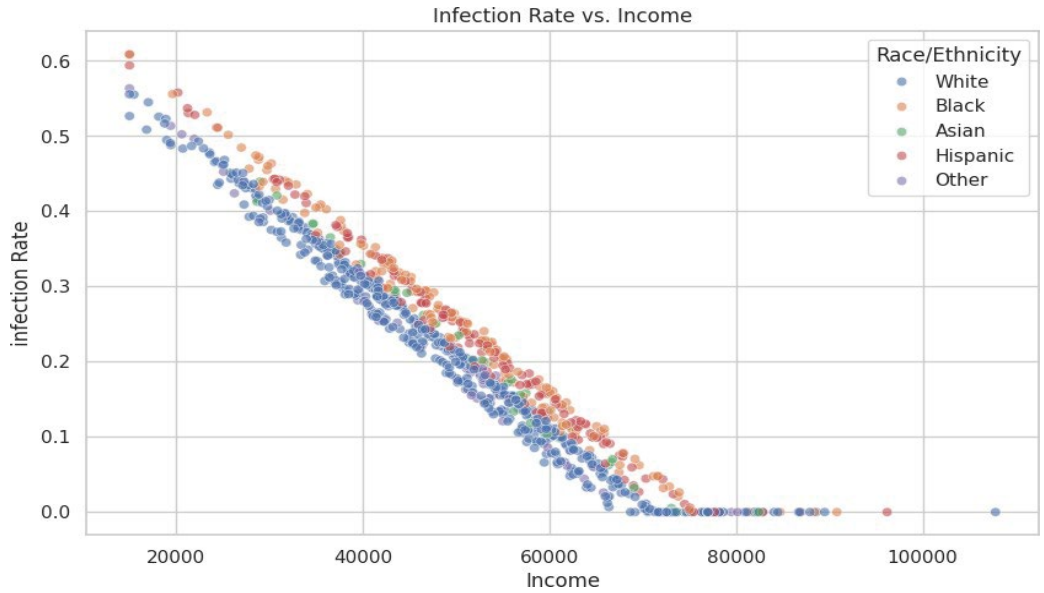

Figure 4: Infection rate by Income.

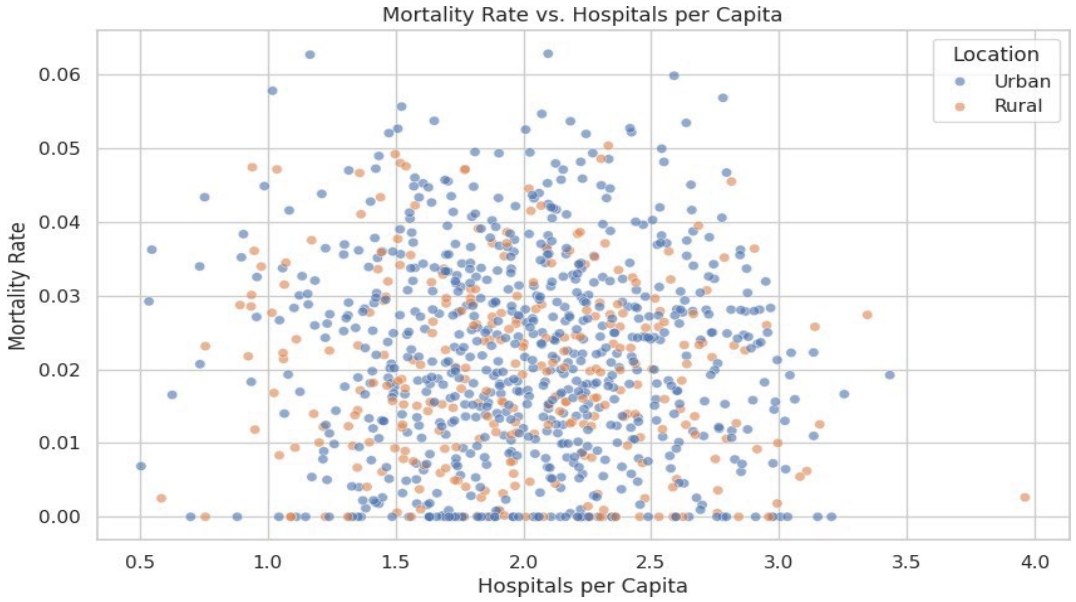

Figure 5: Mortality rate and Hospital per capital.

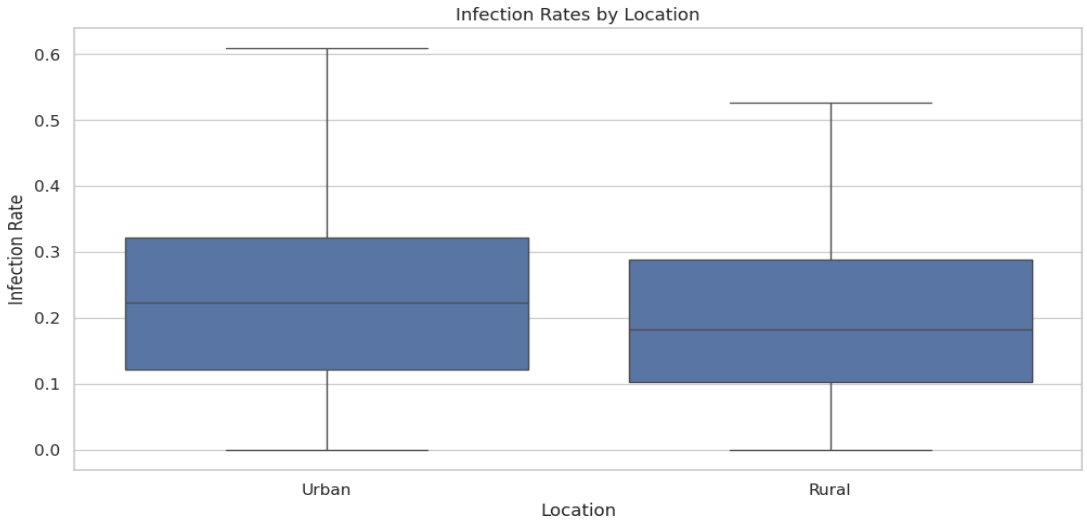

Figure 6: Infection rate by location.

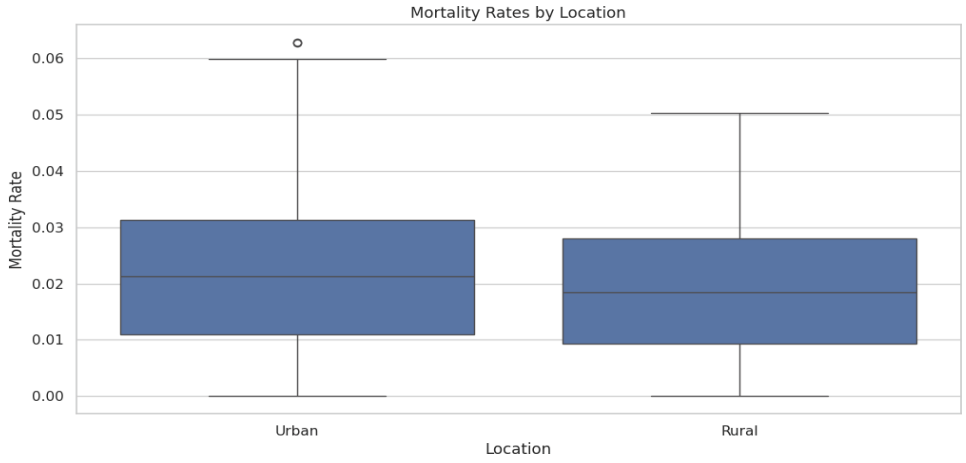

Figure 7: Mortality rate by location.

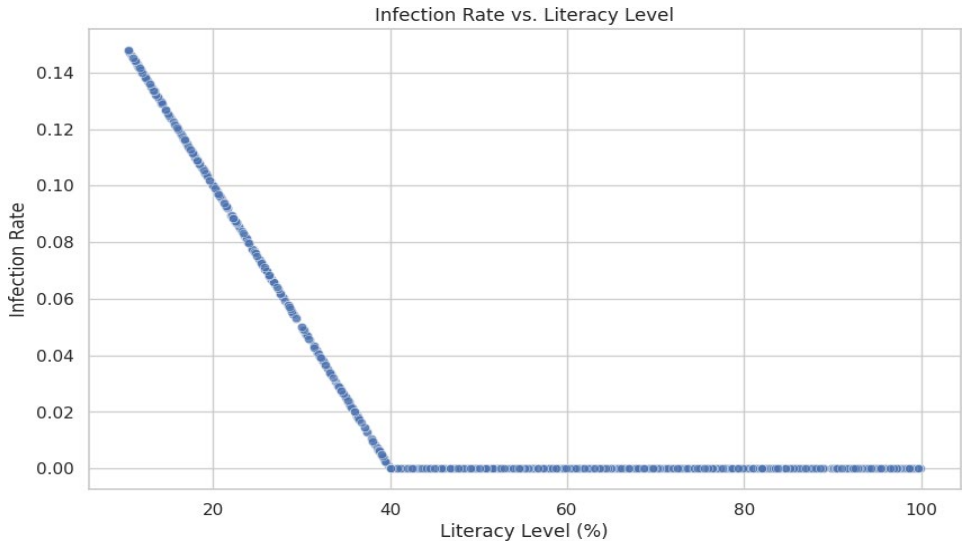

Figure 8: Infection rate and literacy level.

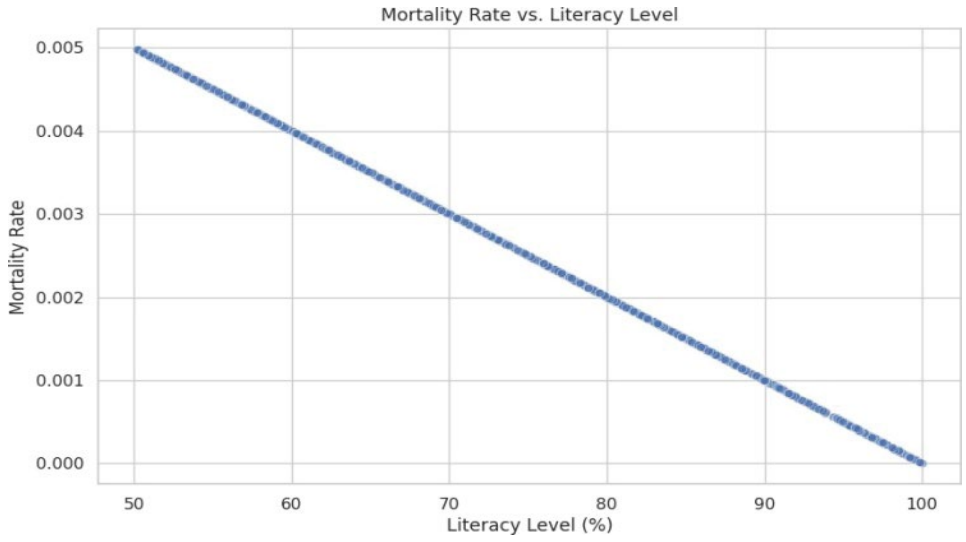

Figure 9: Mortality rate and literacy level.

to protective measures. Similar to scenario in Figure 6, Figure 7 showed that there was inequalities in the infection rate of the regions, Urban region has higher infection rate.

Figure 8 and Figure 9 showed the infection rate and mortality rate respectively as against literacy level. The negative correlation here suggests that higher literacy levels are associated with lower infection rates. Higher

literacy likely leads to better understanding and adherence to public health guidelines, improving prevention efforts. This clearly showed that if the literacy level increases considerably, the mortality will be greatly reduced. Higher literacy levels contribute to better health outcomes by enhancing public understanding of and compliance with health measures. Educational initiatives that improve health literacy can be effective in controlling the spread of infectious diseases. Public health campaigns should be clear, accessible, and distributed through various media to reach all literacy levels.

Conclusions and Recommendations

We analysed and examined inequalities within a non-linear compartmental mathematical model, utilizing integration to assess the effect of hygiene, awareness levels income distribution, education and ethnicity as indicators of social discriminants on controlling the transmission dynamics of COVID-19 within the population under study. The model assumed susceptible individuals come into direct contact with an infected individual thereby contract the infection and indirectly through environmental exposure to the Coronavirus. We also considered the saturation of COVID-19 education across the population over time and the interruption of the transmission rate by improved hygiene practices. Initially, we conducted a qualitative analysis of the deterministic inequalities of non-linear compartmental mathematical model, focusing on the positivity of solutions, boundedness, and the basic reproduction number D_0 when individuals were not practicing good hygiene was obtained. Our findings indicated that the disease-free equilibrium is locally and globally stable if $D_0 = 1$ whereas an endemic state exist for $D_0 > 1$. Furthermore, sensitivity analysis of the system's key parameters to the reproduction number D_0 revealed that the probability of infection P_1, the maximum transmission rate Ω_{Max}, and the restriction rate γ are highly sensitive factors. Our study underscores that information propagation regarding good hygiene practices over time induces behavioral changes that significantly reduce the number of quarantined and infected individuals. Based on these findings, we make the following recommendations:

Restriction (ϖ) is identified as a highly sensitive parameter that increases the basic reproduction number (D_0). Therefore, we recommend considering restrictions on all borders to control entry into the country during outbreaks. These restrictions can vary in intensity depending on the severity of disease spread.

The sensitivity analysis highlights that infection probability increases significantly with direct contact with infected individuals. Thus, strict adherence to measures such as stay-at-home policies, social distancing, and regular hand washing with alcohol-based sanitizers is crucial. These actions effectively interrupt virus transmission, as the virus requires a medium to spread.

Given the high sensitivity of the maximum transmission rate Ω_{Max}, it is imperative to educate the public extensively on personal and community hygiene. Increased awareness through widespread campaigns fosters behavioral changes that can mitigate transmission rates effectively.

Our study reveals that promoting good hygiene practices through education and awareness initiatives can induce attitudinal changes among individuals. Encouraging people to adopt healthy habits protects them against the disease and contributes to public health efforts.

Information dissemination tends to degrade over time due to factors like complacency and resource limitations. Therefore, sustained educational campaigns on disease transmission prevention, utilizing platforms such as social media, TV, radio, and talk shows, are essential.

Implementing these recommendations comprehensively by all stakeholders can potentially flatten the curve of COVID-19 infections. The analysis of our inequality model demonstrates that factors such as hygiene levels, awareness, and healthcare access significantly influence infection and mortality rates. Addressing these disparities necessitates robust policies that support financially, enhance healthcare infrastructure, promote educational initiatives, and tackle systemic inequalities. By doing so, public health responses can become more equitable and effective, thereby improving health outcomes across all populations.

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Data Availability

There is no data used here. The research used simulated data to create different situation during the outbreak of disease.

Ethical Approval

No need of ethical approval in this research paper.

Declaration of Conflict of Interest

We confirmed that there is no conflict of interest among the authors.

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